Basal Mass Balance beneath the GISMO Mosaic

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The volume continuity equation expressed in terms of input (Q_i) and output (Q_o) fluxes is

$$\frac{dh}{dt} = \frac{-(Q_o - Q_i)}{A} + \dot{a} + \dot{b}$$

Here, H is the ice thickness (or surface elevation if the basal interface is fixed), A is the area of a flowband, a is the surface accumulation rate and b is basal freeze rate. The input and output flux gates are choosen orthogonal to the velocity field. The lateral bounds of the flowband are choosen to be flow lines so that there is no flux across the lateral boundaries.

The fluxes are given as

$$Q_f = \sum_{n=1}^{k} \alpha H_n V_n \Delta S_n$$

Here f represents either the flux across the input or output gates, Hn is the ice thickness and Vn is the speed orthogonal across a small element ΔS_n. Vertical variation in the speed is accounted for by a weighting parameter α.

For the mth measurement interval, the flux is

$$Q_m^f = \alpha H_m V_m \Delta S_m$$

Assuming the errors on each measurement parameter are normally distributed, the variance of the flux estimated for the mth interval is

$$\left(\delta Q_m^f\right)^2 = (\alpha V_m \Delta S_m \delta H)^2 + (\alpha H_m \Delta S_m \delta V)^2 + (H_m V_m \Delta S_m \delta \alpha)^2$$

The variance of the flux across the flow gate is

$$(\delta Q^f)^2 = \sum_{m=1}^{n} \left(\delta Q_m^f\right)^2$$

The flux difference across the flow band is simply

$$\Delta Q = Q^{out} - Q^{in}$$

and the variance of the difference between the input and output flux is

$$(\delta \Delta Q)^2 = \left(\delta Q^{in}\right)^2 + (\delta Q^{out})^2$$

An equivalent, average thickening rate is found by dividing the flux diffence by the area

$$\left.\frac{dh}{dt}\right|_{flux\ difference\ only} = \frac{Q_o - Q_{in}}{A} = \frac{\Delta Q}{A}$$

The error on the equivalent thickening rate due to flux imbalance is

$$\delta \frac{dh}{dt} = \left[\left(\frac{\Delta Q}{A} \delta \Delta Q\right)^2 + \left(\frac{\Delta Q}{A^2} \delta A\right)^2\right]^{\frac{1}{2}}$$

Errors in the length of the measurement elements are taken as zero.

There may also be systematic errors in the ice thickness and velocity data. Systematic errors are estimated in two ways. First,

$$\delta \Delta Q|_{systematic} = \sum (\alpha V_m \Delta S_m \delta H)_{out} + (\alpha H_m \Delta S_m \delta V)_{out} - (\alpha V_m \Delta S_m \delta H)_{in} - (\alpha H_m \Delta S_m \delta V)_{in}$$

where the δ terms now indicate a systematic bias in a measurement parameter. Here the same systematic errors are assumed to apply to both gates. Alternatively, a worst case systematic error is estimated assuming that the systematic errors for each gate are different. Here, the magnitude of the error is kept constant but the sign of the error is changed on one of the gates. Then

$$\delta\Delta Q|_{systematic\ by\ gate} = \sum(\alpha V_m \Delta S_m \delta H)_{out} + (\alpha H_m \Delta S_m \delta V)_{out} + (\alpha V_m \Delta S_m \delta H)_{in} + (\alpha H_m \Delta S_m \delta V)_{in}$$

Assuming errors in the flowband area are purely random, then the systematic error in the thickening rate becomes

$$\delta \frac{dh}{dt} = \left(\frac{\Delta Q}{A} \delta\Delta Q|_{systematic}\right)$$

Where choice of the systematic error on the flux difference depends on the likely variation in systematic error across the scene.

Basal topographic relief across the July 2008 study area is shown in figure 1. Relief is overlain by 2004 surface velocity directions (I. Joughin personal comm. and BPRC website). Several, automatically-drawn flow lines trend roughly east west across the base. Two flux gates at either end of the area are drawn orthogonal to the flow direction as illustrated in figure 4. The area of the flow band is 61.6 sq km..

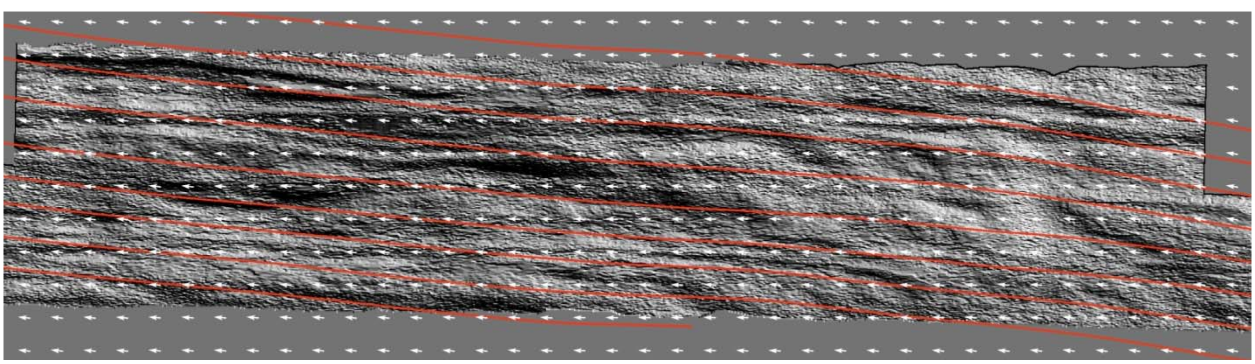

Figure 1. Basal shaded relief overlain with surface velocity direction arrows, manually estimated flow lines and two flux gates at either end of the mosaic.

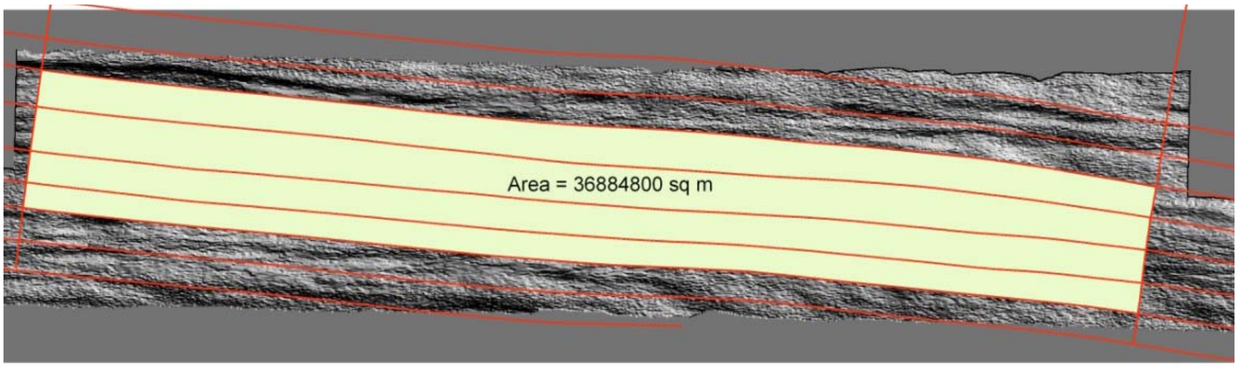

Figure 2. Flow band selected for analysis

The ice thickness across the input and output gates is shown in figure 3

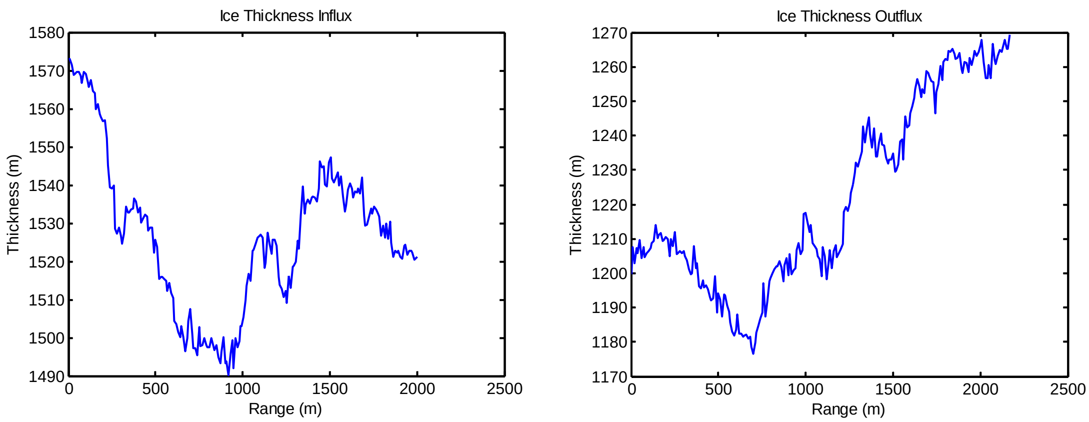

Figure 3. Ice thickness extracted from the GISMO data set along the input (left) and output (right) flux gates. Profiles trend from north to south.

The speed across each gate is shown in figure 4

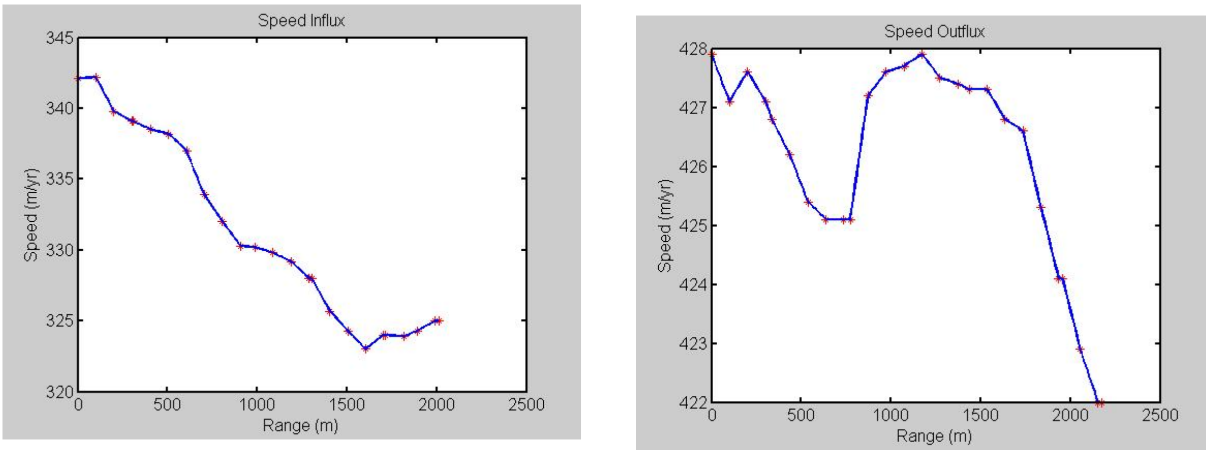

Figure 4. Red stars represent measured speeds. Blue line represents speeds interpolated to locations of ice thickness measurements. Profiles trend from north to south.

Taking $\alpha = 1$, the flux measured across each interval between ice thickness measurements is shown in figure 5. Abrupt drops in flux are those few points where the sampling interval (normally about 10 m) was smaller which is a function of the image processing tool (ERDAS Imagine).

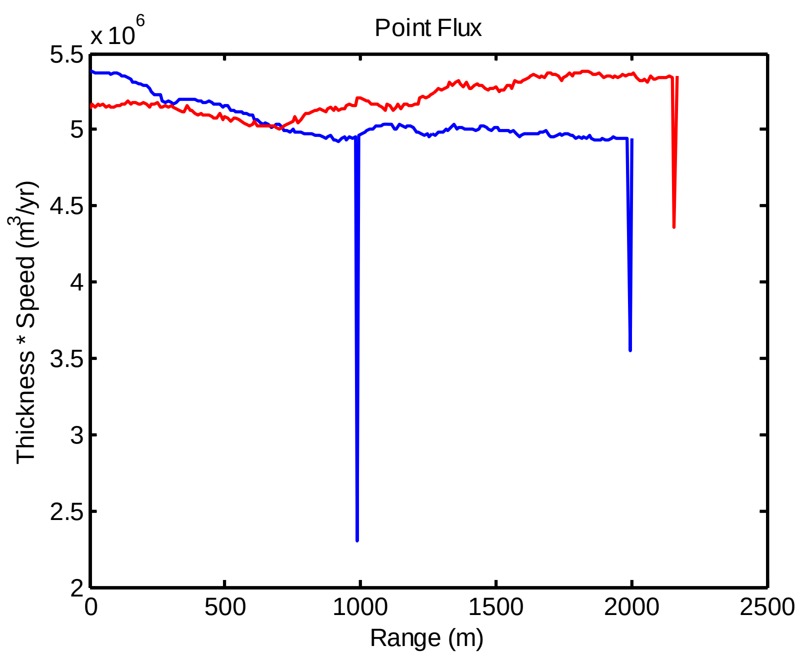

Figure 5. Estimates of input flux (blue) and output flux (red) across each measurement segment along the distance along the flow gates. Profiles trend from north to south.

The total input and output fluxes are found by summing the flux from each interval. Taking α to be unity, the error in the thickness to be 50 m and the error in the velocity to be 50 m/yr, the input flux is 1.018 +/- .01 km^3/yr and the output flux is 1.135 +/- .01 km^3/yr. The flux difference is 0.117+/- .002 km^3/yr. Assuming that there is substantial basal drag (α=0.8), the input flux is 0.8143 +/- .009 km^3/yr and the output flux is 0.908 +/- .008 km^3/yr. The flux difference is 0.0933+/- .01 km^3/yr.

For vertically invariant velocity and a 5% error on the flowband area, the equivalent thickening rate is -3.16 +/- 0.4 m/yr including both the random and systematic errors, noting that this value is only associated with the flux difference. Assuming substantial shear at the base of the ice (α=.8) the differential flux thinning rate is estimated to be -2.50 +/- 0.3 m/yr. Worst case systematic biases result in errors in excess of 10 m/yr.

A second, shorter flow segment was selected to avoid suspected noisy data on the northwestern edge of the ice thickness data set (figure 6)

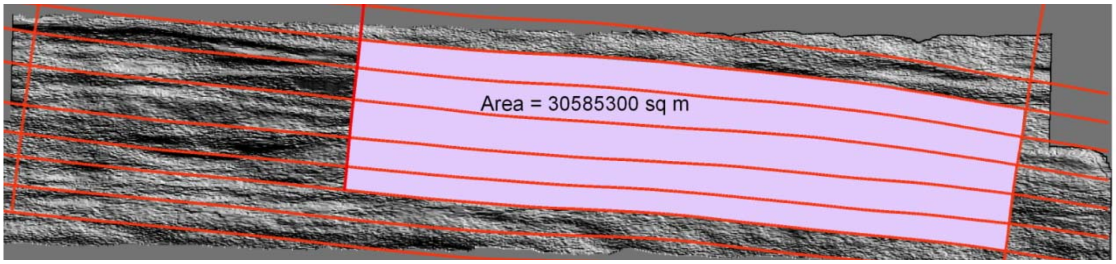

Figure 6. Second flow band which avoids potentially noisy data in upper left sector of the ice thickness data set.

The point flux measurements are shown in figure 7.

Figure 7. Input (blue) and output (red) point flux measurements for the second flowband. Data are ordered north to south.

Again, the total input and output fluxes are found by summing the flux from each interval. Taking α to be unity, the error in the thickness to be 50 m and the error in the velocity to be 50 m/yr, the input flux is 1.30 +/- .01 km^3/yr and the output flux is 1.34 +/- .01 km^3/yr. The flux difference is 0.0365+/- .02 km^3/yr. Assuming that there is substantial basal drag (α=0.8), the input flux is 1.0419 +/- .01 km^3/yr and the output flux is 1.0711 +/- .009 km^3/yr. The flux difference is 0.0292+/- .0130 km^3/yr.

For vertically invariant velocity and a 5% error on the flowband area, the equivalent thickening rate is -1.19 +/- 1.1 m/yr including both the random and systematic errors, noting that this value is only associated with the flux difference. Assuming substantial shear at the base of the ice (α=.8) the differential flux thinning rate is estimated to be -.96+/- 0.88 m/yr. Worst case systematic biases result in errors in excess of 10 m/yr.

Airborne laser altimeter data (W. Krabill) show that the thinning rates in the study area are about -1.5 m/yr (2002-2005). Allowing for basal shear, the balance equations become for the long flow band

-1.5 +-0.5 m/yr = -(2.5 +-0.5 m/y) + interface fluxes where errors on the measured dh/dt are based on local variability in the measurements.

Short band

-1.5+-0.5 m/yr = -(0.96 +-0.88) + interface fluxes

Taking an average of the two bands

-1.5 +- 0.5 m/yr = -(1.74 +- 0.5 m/yr) + interface fluxes

Given that the study area is close to the equilibrium line (surface accumulation estimated to be about 10 cm/yr from Box 2006), the implication is that thinning is dynamically controlled. The data are not sufficiently accurate to estimate a basal melt rate. Formally, the melt rate is not statistically significant from zero in this region.